Throughout, assume that $m$, $n$, and $p$ are positive integers.

**Definitions**

You should be able to provide *precise* definitions of any and all of the following. Throughout, assume that $m$, $n$, and $p$ are positive integers.

- $\mathbb{R}$, $\mathbb{R}^n$, $\mathbb{M}_{m \times n}(\mathbb{R})$, $\mathbb{M}_n(\mathbb{R})$, $0$ in $\mathbb{R}^n$, and $I_n$ in $\mathbb{M}_n(\mathbb{R})$;
- What it means for a linear system to be *consistent* or *inconsistent*;
- What it means for two linear systems to be *equivalent*;
- What it means for two matrices to be *row equivalent*;
- What it means for vector $v$ to be a *linear combination* of vectors $v_1, v_2, \ldots, v_p$ in $\mathbb{R}^n$;
- The *span* of vectors $v_1, v_2, \ldots, v_p \in \mathbb{R}^n$.
- The *dot product* $u \cdot v$ of vectors $u = (u_1, \ldots, u_n)$ and $v = (v_1, \ldots, v_n)$ in $\mathbb{R}^n$.

In addition, here is a summary of things you should know for the exam.

**Vectors, Matrices, and their Operations**

- Know that a matrix being $m \times n$ means that it has $m$ rows and $n$ columns.
- Know that if $v$ is in $\mathbb{R}^n$, then $v$ is said to have dimension $n$.
- Know how to add vectors in $\mathbb{R}^n$ and how to multiply them by scalars. Know that the vector sum $v + w$ exists only when $v$ and $w$ have the same dimension.
- Know the algebraic properties of $\mathbb{R}^n$ under addition and scalar multiplication (see the *Algebraic Properties* handout). You won’t have to state these, but you must be able to use them.
- Know two ways of computing $Ax$, given $A \in \mathbb{M}_{m \times n}(\mathbb{R})$ and $x \in \mathbb{R}^n$. Know that $Ax$ is only defined when the number of columns of $A$ and the number of rows of the column vector $x$ are equal.
- Know that if $A \in \mathbb{M}_{m \times n}(\mathbb{R})$, $u, v \in \mathbb{R}^n$, then $A(u + v) = Au + Av$. 
• Know how to correctly denote the multiplication of vector by a constant (the constant must be written to the left of the vector), and the multiplication of an \( m \times n \) matrix by a vector in \( \mathbb{R}^n \) (the matrix must be written to the left of the vector).

**Solving Linear Systems, Vector Equations, and Matrix Equations**

• Know how to represent (1) a linear system of \( m \) equations in \( n \) variables, \( x_1, x_2, \ldots, x_n \) using

  (2) an augmented matrix \([A|b]\);

  (3) a vector equation \( x_1v_1 + x_1v_1 + \cdots + x_pv_p = b \); and

  (4) a matrix equation \( Ax = b \).

Know how to translate back and forth between these four ways of representing such a system.

• Know how to recognize when a given matrix (augmented or not) is in REF and/or RREF.

• Given a matrix, know how to reduce it to a REF matrix or its RREF

  (1) manually, using elementary row operations (that is, using the RRA/Gaussian elimination);

  (2) using your calculator.

• Know that any linear system has 0, 1, or infinitely many solutions.

• Know how to identify the the pivot positions in any matrix.

• Given a linear system, vector equation, or matrix equation:

  ◦ Know how to determine whether it has any solutions (that is, whether or not the corresponding linear system is consistent or inconsistent), and determine how many solutions it has, by looking at a REF of an augmented matrix.

  ◦ Know how to determine its free variables and dependent variables by reducing an augmented matrix to REF.

  ◦ Know how to solve it by reducing an augmented matrix to RREF.

  ◦ Know how to correctly write its solution set.

In cases in which it has infinitely many solutions, know how to write its solution set—that is, its general solution—using a parametric description, and know how to identify some of its particular solutions (your choice).

TURN OVER
Linear Combinations and Spans

- Know that the span of a set of vectors contains either infinitely many elements, or only the zero vector, \( \mathbf{0} \).

- Know how to determine if a given vector \( \mathbf{b} \) in \( \mathbb{R}^n \) is a linear combination of given vectors \( \mathbf{v}_1, \ldots, \mathbf{v}_p \) in \( \mathbb{R}^n \). (Equivalently: know how to determine if a vector in \( \mathbb{R}^n \) is in the span of given vectors \( \mathbf{v}_1, \ldots, \mathbf{v}_p \) in \( \mathbb{R}^n \).) If the given vector \( \mathbf{b} \) is such a linear combination, know how to find weights \( c_1, \ldots, c_p \in \mathbb{R} \) such that \( \mathbf{b} = \sum_{i=1}^{p} c_i \mathbf{v}_i \).

- Know the Span Theorem.

- Know how to determine if a given set of vectors \( \mathbf{v}_1, \ldots, \mathbf{v}_p \) in \( \mathbb{R}^n \) spans \( \mathbb{R}^n \). (Hint: use the fourth equivalent condition in the Span Theorem.)

- Know that \( n \) vectors cannot span \( \mathbb{R}^m \) if \( n < m \).

  But also note that having \( n \geq m \) vectors in \( \mathbb{R}^m \) is NOT a sufficient condition for them spanning \( \mathbb{R}^m \). Can you give an example of, for instance, 3 distinct vectors in \( \mathbb{R}^2 \) that don’t span \( \mathbb{R}^2 \)?