Proof Without Words: Rearranged Alternating Harmonic Series

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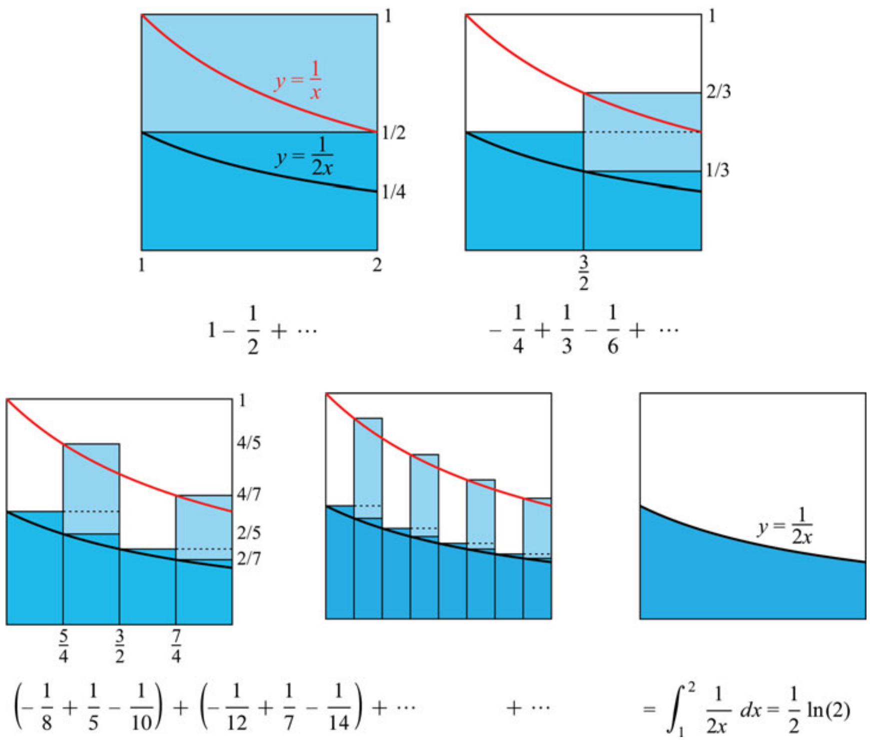
Lord Brouncker [1], and more recently Hudelson [2], visually demonstrate that the alternating harmonic series converges and has sum equal to ln(2). We modify their technique to demonstrate that

$$1 - \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{6} - \frac{1}{8} + \frac{1}{5} - \frac{1}{10} - \frac{1}{12} + \frac{1}{7} - \frac{1}{14} + \cdots = \frac{1}{2}\ln(2).$$



Summary. We visually compute the sum of a rearranged alternating harmonic series.

References

[1] Brouncker, W. (1668). The squaring of the hyperbola, by an infinite series of rational numbers, together with its demonstration. *Phil. Trans.* 3: 645–649. doi.org/10.1098/rstl.1668.0009.

[2] Hudelson, M. (2010). Proof without words: The alternating harmonic series sums to ln(2). *Math. Mag.* 83: 294. doi.org/10.4169/002557010x521831.

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