Proof Without Words: Sums of Powers of $\frac{4}{9}$

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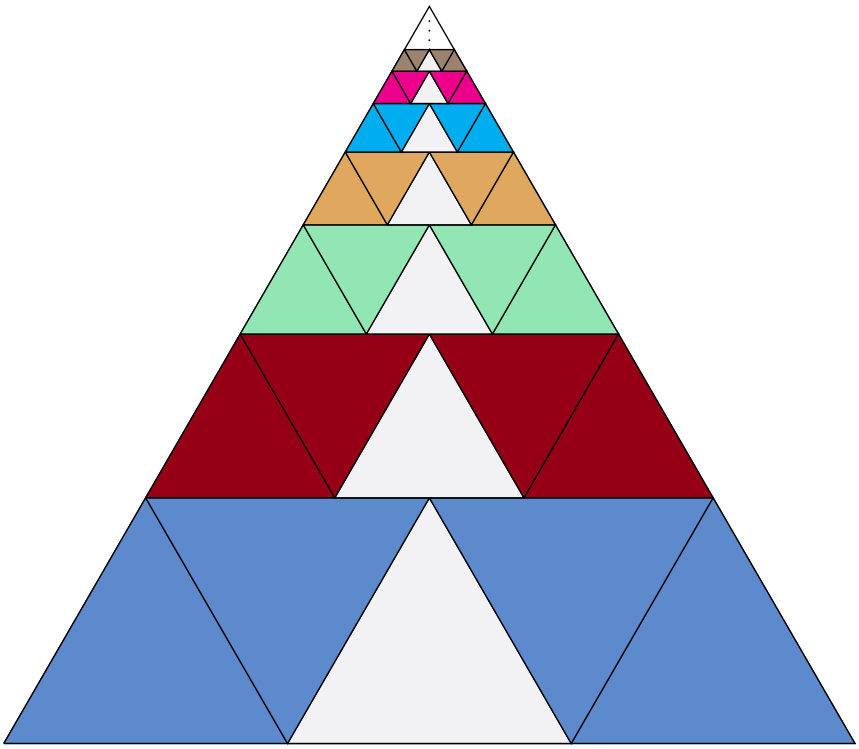
Mabry [**2**] used a partition of an equilateral triangle into four similar equilateral triangles to show that $\frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \cdots = \frac{1}{3}$. We use a similar approach to show that $\frac{4}{9} + \left(\frac{4}{9}\right)^2 + \left(\frac{4}{9}\right)^3 + \cdots = \frac{4}{5}$.



In [**1**], it is shown that an equilateral triangle can be decomposed into n equilateral subtriangles (as long as $n \neq 2, 3$ or 5); is it possible to determine which other series allow an analogous proof without words?

REFERENCES

1. R. W. Freese, A. K. Miller, Z. Usiskin, Can every triangle be divided into n triangles similar to it?, *Amer. Math. Monthly* **77** no. 8 (1970) 867–869.
2. R. Mabry, Proof without words: $\frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \cdots = \frac{1}{3}$, *Math. Mag.* **72** no. 1 (1999) 63.

Summary. We provide a visual computation of a particular infinite series.

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