ℕUMS 2013 Abstracts

Invited Address

Inga Johnson, Willamette University

Topology, Homology and Applications to Data

Topology is the subfield of mathematics that is concerned with the study of shape. Mathematicians have studied topological questions for the past 250 years. However, in just the past 15 years topology has been found to have many different applications to real world problems. One of these is to use a topological tool called persistence homology to understand and analyze high dimensional and complex data sets.

This talk will be an introduction to topology and the concept of homology. We will then use homology to a look at examples of how topological ideas can be used to give new and surprising insight towards understanding data.

Student Speakers

Christopher Anderson, Lewis & Clark College

Biological Implications of the Mathematical Properties of Ciliate Genome Decryption.

Ciliates are a fascinating family of protozoa which store their genetic information in an encrypted micronucleus - before any ciliate gene can be expressed, it must be unscrambled based on a set of genetic "pointers" and copied into a decrypted macronucleus. Representing the scrambled micronuclear DNA as integer permutations of the canonical gene sequences reveals that ciliate molecular machinery is capable of inverting certain permutations. Though we cannot yet precisely describe the set of ciliate-invertible permutations, we have observed optimal patterns for the decryption mechanism, leading to specific hypotheses about the permutations possible in real-world ciliate DNA. In this talk I present some of our findings.

Daniel Baron, Western Washington University

Straight-edge and Compass: Constructing the Heptadecagon

The problem of constructible polygons fascinated the ancients, who discovered the methods for constructing the regular 3-, 4-, and 5-gon. Then progress stalled for 2,000 years, until Gauss proved something so exciting that he wanted it on his tombstone. I will demonstrate Gauss's result and perform a construction of the regular 17-gon.

Kyle Brooks, University of Puget Sound

Numerical Techniques in Bioeconomic Optimal Control

The field of bioeconomics is concerned with the optimal management of natural resources. Because natural resource levels generally change over time, solutions to these management problems usually assume the form of time-dependent control functions. The mathematical theory of optimal control theory covers a range of analytical and numerical techniques that can be used to produce such control functions when the time evolution of the resource is given by a differential equation. Example applications include the management of fisheries and forests, as well as predator-prey models with varying control systems.

On a numerical level, techniques commonly used to solve these problems include a forward-backward sweep method and a linear discretization method that solves a non-linear program. There are also methods that impose additional restraints on the control function, such as forcing it to be piecewise constant with a discrete number of jumps. This talk examines the effectiveness of these techniques in solving a particular class of problems that emerge from fisheries management.

Edward Charlesworth, Seattle University
Pressure along Streamlines

In this talk, the derivative of the pressure along a particles streamline is analytically derived from Euler's equations in a traveling frame. The derivation requires use of a Hodograph transformation and shows that the pressure is not constant along the streamline. Pressure distributions are numerically calculated along all streamlines under the wave. This is a early step in relating the pressure along a streamline to the wave height. We wish to develop a new method to measure wave-height using pressure measurements internal to the fluid domain.

Eli Cohen and Olivia Nebeker, Eastern Oregon University
Introduction to Partial Inertia Tables and an Example

This presentation gives a basic introduction to the inertia tables of graphs with symmetric zero/non-zero adjacency matrices. This will be an overview of the terminologies and concepts involved in this method for finding the signs of the eigenvalues a given graph's set of adjacency matrices can attain. We will also work through an example of how to apply the techniques we have been studying at Eastern Oregon University in La Grande, Oregon.

Weston Cox, Seattle University
Modeling Owls' Sound Location Methods

This project involved modeling owls' methods to track prey. This included creating a dynamical system to represent a mouse's erratic movements, as well as exploring methods to model the owl's capabilities to track the mouse. We will compare the owl's tracking of prey with the performance of an optimal tracking algorithm implemented using particle filtering. These results will be compared with previous research performed on owls and their abilities to place sound sources in space using the associated Interaural Time Difference.

Sarah Erickson, Western Washington University
Secondary Aneurysm Formation due to the Effects of a Primary Aneurysm

Intracranial aneurysms are localized dilations of arterial vessels located around the Circle of Willis, an important network of arteries at the base of the brain. Aneurysms are at constant risk of hemorrhage; however, the number of benign cases carried by the populace, the dangers of treatment, and the risk of recurrence often null the efficacy of preventative surgery. Although the mechanisms behind the formation of individual intracranial aneurysms have been thoroughly modeled as the consequence of local hemodynamic conditions, previous simulations have concentrated on single aneurysms. Using OpenFOAM, an open source fluid dynamics toolkit, we model how changes in the hemodynamics within the Circle of Willis caused by the presence of a primary aneurysm can facilitate the formation of a secondary aneurysm. We determined the change in risk of developing a secondary aneurysm on the anterior communicating artery given a primary aneurysm at the bifurcation between the posterior communicator artery and the basilar artery. We found an average decrease in wall shear stress at the anterior communicating artery of 0-4%. Further, the necessary theory behind modeling turbulent flows with the Reynolds-Averaged Navier-Stokes equations is exposited.

Eric Herde, Pacific Lutheran University
A Mathematical Approach to Finding a Fair Election System

What does Arrow's Theorem actually say about elections? Is it actually possible to have a fair election system? This presentation expands upon my previous research in this area and investigates how one such system would operate in legislative elections in Washington.

Sky Hester, Western Washington University
Counting with Symmetry: Polya-Redfield Enumeration of Nonisomorphic Graphs

How many essentially different ways can the faces of a die be labeled with the numbers one through six? How many essentially different graphs exist of a particular size and order? A surprising connection between group actions and generating functions discovered independently by Redfield in 1927 and by Polya ten years later not only provides answers to these questions and others like them, but beautifully illuminates their hidden symmetry. A highly visual explanation of the solution to the problem of enumerating nonisomorphic graphs of all orders and sizes with the Polya-Redfield method will be presented.

Vinay Iyengar, Oregon Episcopal School
Efficient Characteristic 3 Galois Field Operations for Elliptic Curve Cryptographic Applications

Galois fields are a fundamental concept in abstract algebra and have a wide variety of applications in applied mathematics and computer science. Galois fields of characteristic 3, where the number of field elements is a power of 3, have a distinctive application in building high-security elliptic curve cryptosystems. However, they are not typically used because of their relative inefficiency in computing polynomial operations when compared to conventional prime or binary Galois fields. The purpose of this research was to design and implement characteristic 3 Galois field algorithms with greater overall efficiency than those presented in current literature, and to evaluate their applicability to elliptic curve cryptography. The algorithms designed were tested in a C++ program and using a mapping of field element logarithms, were able to simplify the operations of polynomial multiplication, division, cubing, and modular reduction to that of basic integer operations. In exchange for the initial precomputation and storage costs, the algorithms designed significantly outperformed the best characteristic 3 algorithms presented in literature. Furthermore, they performed elliptic curve scalar multiplication with an efficiency comparable to prime Galois fields and showed a distinct applicability to elliptic curve cryptosystems with constrained environments or large-scale storage potential. In conclusion, this research presents a novel method of optimizing the performance of characteristic 3 Galois fields and has major implications for the field of elliptic curve cryptography.

Daniel Juda, Pacific Lutheran University
Dominance on $\mathbb{N}$

Famous theorems due to Lucas and Kummer are deeply connected to the study of Pascal's Triangle mod p for p prime. We discuss these connections by studying a class of partial orders on $\mathbb{N}$, called b-dominance orders, denoted $\ll_b$. On the surface, the innocent definition of these orders may lead one to believe that the orders are unremarkable; yet, we found fascinating connections between the combinatorics of these partial orders and number theory. In particular, we defined a new class of sequences and their associated generalized binomial coefficients. These coefficients, which turn out to be integral, along with the rank function for b-dominance allow us to prove a (previously unknown) analog of Kummer's Theorem for non-prime numbers. Moreover, we investigate more deeply the connection between dominance orders and these generalized binomial coefficients mod b.

Hanna Landrus, Pacific University
Exploring Infinite Sequences of Hitches

A hitch is a tangle about a pole. When climbing a hitch could save a climber's life; when sailing a hitch could prevent a boat from floating away. In this talk we will present a model for determining when a hitch will hold, in particular we will relate the determinate of a matrix to the friction required for the hitch to hold. We will then use this model to further explore hitches. This exploration includes sequences of hitches, as well as hitches with a restricted number of crossings. We will explore two sequences of hitch and how the minimum friction to hold is affect as the pattern in these hitches is repeated infinitely. We will show that the determents resulting from one of our hitch sequences has a closed form that relates to Pascal's Triangle.

Remy Levin, Western Washington University
Dimension, Hutchinson Operators, and Self-Similarity: fundamental concepts in fractal geometry

Fractals have often captured the popular imagination, but what are they really? In this talk I will cover some elegant results in fractal geometry that help to illuminate the true nature of self-similar fractals like the Cantor set and the Koch curve. We will discuss what it means for a set to have a "fractional" dimension, and define Hutchinson operators, an elegant way to describe, mathematically, these complicated objects. If time allows we will look at a beautifully simple way to calculate the dimension of self-similar fractals using Hutchinson Operators.

Nam Nguyen and J. Gage Winde, Eastern Oregon University
Inertia Tables for Families of Undirected Simple Graphs

This presentation consist of a few results, discovered and proved during the 2012 summer research group at Eastern Oregon University. In particular we focus on three families of graphs that we investigated over the summer. Inertia tables are a visual representation of the possible inertias held by a given graph. The inertia of a graph counts the number of positive real eigenvalues and negative real eigenvalues of its corresponding adjacency matrix. The problem of studying inertia tables is directly related to the inverse eigenvalue problem and can be used as a tool for the minimum rank problem.

Timothy Perisho, Seattle Pacific University
Generalized Ramsey Numbers for Near-Diagonal Pan Graphs and Relatively Prime Tadpole Graphs

This paper uses the known exact Ramsey number formulas for arbitrary pairs of cycle graphs as well as several original methods to prove exact Ramsey number formulas for some non-trivial tadpole graphs, from which cycle graphs form the trivial subset. A tadpole graph Qn,t is simply a cycle of size n with a tail of size t, i.e. a path graph Pt connected from one of its endpoints to a point in the cycle Cn. A pan graph Qm is defined as a tadpole graph with t=1. We prove exact formulas for all near-diagonal pan graphs, i.e. those pairs of pan graphs where the maximum cycle size of the pair is no more than 5/2 times the minimum. Then, we demonstrate a general number-theoretic method to find exact Ramsey numbers for near and arbitrarily-far-from-diagonal tadpole graphs that meet certain constraints of relative primality, as well on parity of orders and indices in a relevant modulo arithmetic. We demonstrate that these scattered results provide bounds for other tadpole graph pairs between them that fail to meet the constraints (including far-from-diagonal pan graphs). Finally, we develop an algorithm for finding cases that meet the constraints and give a table of small cases.

Casey Pinckney, Seattle University
The Combinatorial Structure of PS-Ear Decomposable Graphs

A graph is PS-ear decomposable if it can be decomposed as a union of a short cycle and short paths. In this talk, we will present a canonical way in which to represent a convex ear decomposable graph that will make its combinatorial structure easier to understand. This representation gives rise to a natural connection between PS-ear decomposable graphs and families of monomials that encode this combinatorial data. Our work is a first step toward a more general approach of solving a longstanding conjecture about the combinatorial structure of matroids that was made in the late 1970's by combinatorist Richard Stanley.

Brian Schiller, Western Washington University
The Ninja Assassin Wonderwall Game

In this summer camp game, each of n players chooses one person to be their Ninja Assassin and another to be their Wonderwall. When the game starts, each player runs to try to keep their Wonderwall on the line between themself and their Ninja Assassin. We create an algorithm, based on finding cycles in directed graphs, that creates a configuration satisfying every player's constraints or says that no such configuration exists. We also use a computer model to estimate the probability that a given game has such a configuration.

Patrick Sprenger, Seattle University
Stability of Waves with Constant Vorticity

In 2006, Ablowitz, Fokas, and Musslimani reformulated Eulers equations to describe the fully nonlinear dynamics of water waves[1]. Using a generalization of their equation (see [2]), we are able to determine the stability of waves propagating in one spatial dimension with constant vorticity. In this talk, we will discuss the stability of all physically realizable perturbations to traveling wave solutions. In order to accomplish this, we linearize the system about stationary solutions with an arbitrary perturbation of small amplitude and apply Hills method to determine spectral stability of these perturbations [3]. We go through Hills method analytically while the stability computations are done with numerical methods. [1] M. J. Ablowitz, A. S. Fokas, and Z.H. Musslimani, On a new non-local formulation of water waves, J. Fluid Mech., 562 (2006), 313-343. [2] A.C.L Ashton and A. S. Fokas, A non-local formulation of rotational water waves, J. Fluid Mech., 689 (2011), 313-343. [3] Bernard Deconinck and J. Nathan Kutz, Computing spectra of linear operators using the Floquet-Fourier-Hill method, J. Comp. Physics, 219 (2006), 296-321.