## Proof Without Words: Matchstick Triangles

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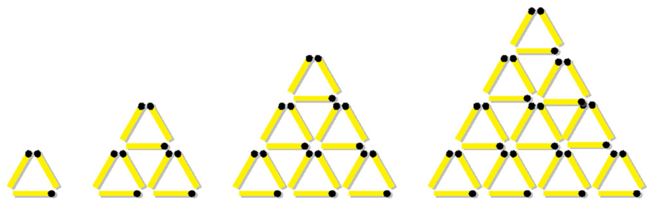
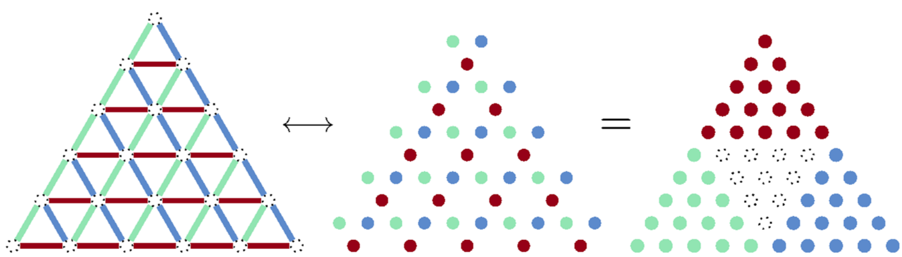
The *triangular matchstick number* $M_n$ counts the number of matches in a triangular match diagram as illustrated below; $M_1 = 3$, $M_2 = 9$, $M_3 = 18$, and $M_4 = 30$.



**Theorem.** *The triangular matchstick numbers satisfy* $M_n = 3T_n = T_{2n} - T_{n-1}$ *where* $T_m = 1 + 2 + \cdots + m$ *is the usual triangular number.*

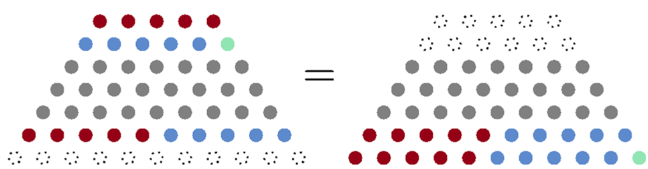
*Proof.*



For instance, $M_5 = 45 = 3T_5 = 3 \cdot 15 = T_{10} - T_4 = 55 - 10$.

**Corollary.** *The triangular matchstick number* $M_n$ *is both the sum of* $n+1$ *consecutive integers and* $n$ *consecutive integers, i.e.,* $M_n = T_{2n} - T_{n-1} = T_{2n+1} - T_{n+1}$.

*Proof.*



**Exercise.** Prove that $M_n$ is the *least* positive integer that is the sum of $n+1$ consecutive positive integers and $n$ consecutive positive integers. (Hint: Assume that there is a positive integer $x$ with $x = na + T_n$ and $x = (n+1)b + T_{n+1}$ such that $0 \leq a < n+1$ and $0 \leq b < n-1$ and obtain a contradiction.)

**Summary.** We define the triangular matchstick numbers and wordlessly prove that each one is three times a triangular number as well as the difference of two triangular numbers.

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