Proof Without Words: Factorial Sums

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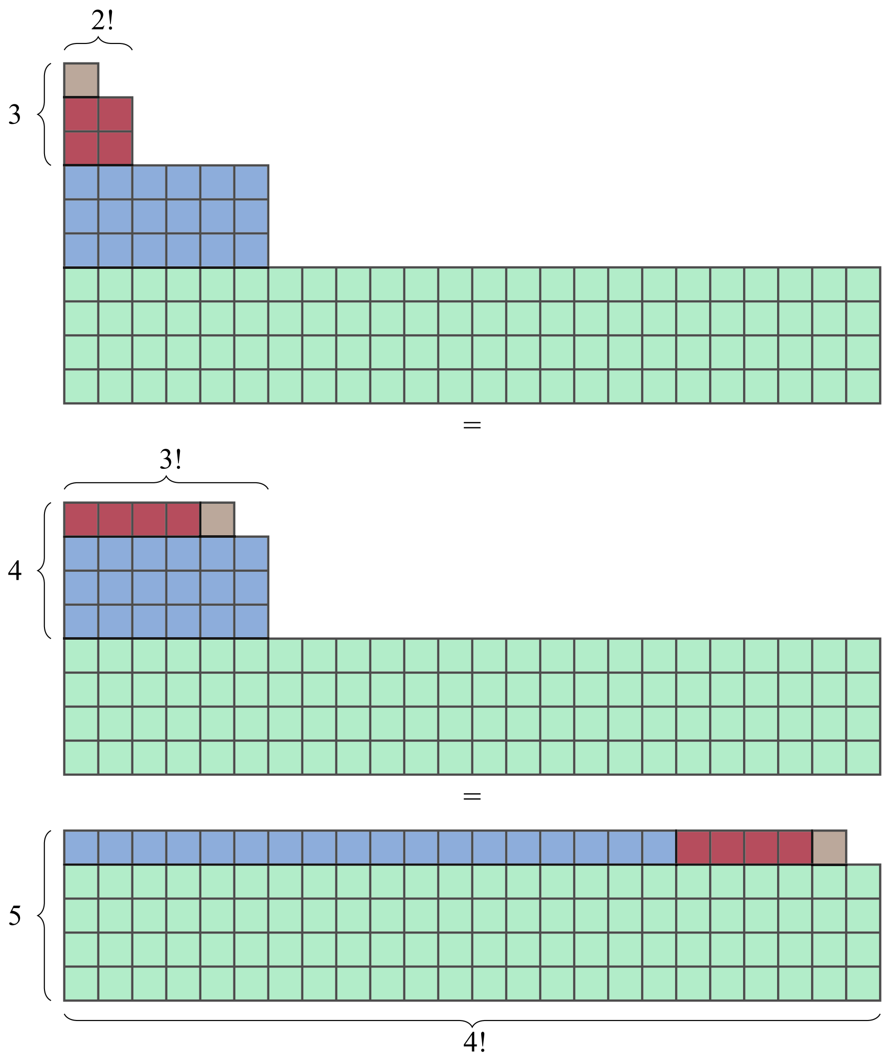
Theorem. *For any integer* $n \geq 1$, *we have* $\sum_{i=1}^{n} i \cdot i! = (n+1)! - 1$.

Proof. (e.g., for $n = 4$).



Corollary. *For each positive integer* m, *there exists a unique sequence of integers* $\{a_i\}_{i \geq 1}$, *satisfying* $0 \leq a_i \leq i$ *for all* i, *such that* $m = \sum_{i \geq 1} a_i \cdot i!$.

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MSC: Primary 05A10

Remark. Given such a representation of m, the theorem describes how to produce a representation of $m+1$; the corollary follows by induction (since $1 = 1 \cdot 1!$).

Summary. We provide a visual proof of a factorial sum identity implying the existence of the factorial base number system.

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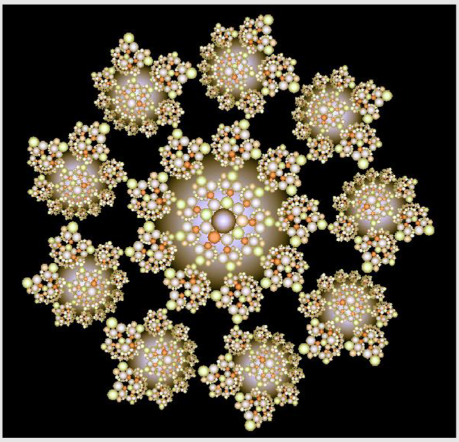

Artist Spotlight
Anne Burns

Nine plus one, Anne Burns; digital print, 2005. An iterated function system made up of Mobius transformations that map the unit disk to a chain of nine disks plus a disk in the center. The nine disks are pairwise tangent, tangent to the unit circle, and tangent to the tenth disk in the center of the unit circle.

See interview on page 375.

L	A	F	F			B	A	B	E		M	A	K	E
I	L	I	A	D		A	R	O	D		A	L	E	S
S	I	L	V	E	R	B	E	R	G		T	I	N	T
A	T	L	A	N	T	A		G	E	O	R	G	I	A
				S	E	R	F			M	I	N	G	
A	N	N	I	E	S		R	O	L	E	X			
L	A	O	S				A	J	O	G		A	M	S
U	P	A	R	R	O	W	N	O	T	A	T	I	O	N
M	A	A		O	P	E	C				I	D	E	O
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	T	W	I	G			S	E	C	T				
N	O	E	T	H	E	R		T	A	A	L	M	A	N
E	T	A	L		C	A	N	T	A	R	E	L	L	A
W	A	V	E		H	I	Y	A		I	N	L	A	Y
S	L	E	D		O	N	E	S			T	E	N	S