basis using an index of performance that incorporates risk. Awards could then be made on a risk-adjusted basis as well as on the absolute highest return. Although such changes would not assure that participants construct well-diversified portfolios, at least participants would become more aware of the importance of portfolio construction and diversification.

NOTES

2. A similar result is reported in Wood, O'Hare, and Andrews (1992).
3. The holding period return is [(ending value/beginning value – 1) × 100%]. If the beginning value is $10 and the ending value is $11, the holding period return is [(11/10 – 1) × 100% = 10%].
4. For a discussion of the efficient market hypothesis, consult an investments textbook such as Bodie, Kane, and Marcus (1989).
5. This analysis of the variability of the returns uses a standard deviation instead of the coefficient of variation because the standard deviation is later used in the Sharpe index of performance to rank portfolios on a risk-adjusted basis.
6. Wood, O'Hare, and Andrews (1992, 242-43) come to a similar conclusion when they discuss diversification as a strategy.
7. The noncomputerized stock market game with a structure that rewards relative performance as well as absolute performance is reported in Bell (1993).
9. A reasonably well-diversified portfolio may be achieved with 15 to 20 securities. See, for instance, Evans and Archer (1968) and Sharpe (1972). Given the short time period of the game, it may be impossible to achieve sufficient diversification. If that is the case, this shortcoming should be made explicit to the participants.
10. The current inclusion of the risk-adjusted performance measures as part of the CFA and CFP professional designations may increase their usage to judge portfolio managers. After the initial creation of risk-adjusted returns, rating services did supply portfolio managers with performance measures, but these measures have never been popular with portfolio managers. One possible explanation is the managers' poor performance, a result that is consistent with the EMH. See the discussion in Bodie, Kane, and Marcus (1989, pp. 724-26).

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easy illustration of Hotelling's rule and gives students an intuitive feel for this important result. However, two-period analysis on its own is not enough to give students a full understanding of the nature of optimal resource depletion. The main limitation is that it does not generate the actual optimal price and extraction paths for given real world situations. What it fails to identify is the optimal starting price, which, when combined with Hotelling's rule, specifies the full optimal price path and (consequently) the optimal extraction path. In addition, two-period analysis cannot indicate how the optimal price and extraction paths will change in response to changes in key variables (such as the stock size). Only a multiperiod-based analysis can generate these more complete, and practically useful, outputs.

The multiperiod solution can be obtained either via mathematical optimization (see Tietenberg 1992, appendix to chap. 6) or alternatively by a graphical approach. The focus here is on the graphical approach. Pearce and Turner (1990, Figure 18.1), present a useful multiperiod graphical exposition using a four-quadrant diagram approach. This presentation suffers, however, from two deficiencies. First, it involves an arguably tedious trial-and-error graphical solution method, where the starting price is iterated until total cumulative extraction, measured by the area under the extraction time path, equals total stock. Each time the starting price is varied, a new extraction time path is derived, and a new cumulative extraction area must be measured. Second, the important impact of discounting on the value of resource extraction is not adequately illustrated.

MODIFIED MULTIPERIOD PRESENTATION

Now consider an alternative improved N-period approach that overcomes these deficiencies. The approach is based on the usual assumptions of identical and stable demand schedules across all time periods and constant marginal cost of extraction (MC). Following Tietenberg (1992), I focus on the key variable, marginal net benefit in time period t, \( MNB_t \), where

\[
MNB_t = P_t - MC_t, \tag{1}
\]

where \( P_t \) is the inverse demand in period \( t \), and is assumed to be linear. That is

\[
P_t = a - bq_t, \tag{2}
\]

where \( a \) and \( b \) are constants, and \( q_t \) is extraction in period \( t \).

Expressing \( MNB_t \) schedules in present value terms, denoted \( PVMNB_t \);

\[
PVMNB_t = \frac{MNB_t}{(1 + r)^{-t}}, \tag{3}
\]

where \( r \) is the discount rate (expressed in units %/100). Substituting equation (2) into equation (1), and (1) into (3) yields

\[
PVMNB_t = \frac{(a - MC_t)}{(1 + r)^{-t}} - \frac{b}{(1 + r)^{-1}} (q_t). \tag{4}
\]

Figure 1 shows \( PVMNB_t \) curves for the present period (period 1) and all future periods. Discounting rotates the \( PVMNB_t \) curves progressively anticlockwise for consecutive time periods, while pivoted at the extraction level \( q_0 \) (the extraction level where \( P_t = MC_t \), thus \( MNB_t = 0 \), thus \( PVMNB_t = 0 \) and is thus independent of \( t \); and from equation (4), it follows that \( q_0 = \frac{(a - MC_t)}{b} \). The higher the discount rate, the greater the rotation in each period. At each level of extraction below \( q_0 \), \( MNB_t > 0 \), and thus \( PVMNB_t \) becomes progressively smaller as each period passes, reflecting the fact that extraction today is preferred to extraction tomorrow by a factor \( (1 + r) \). As \( r \) becomes large, \( PVMNB_t \) approaches but never reaches the horizontal axis. The full impact of discounting on \( MNB_t \) for all time periods is thus captured in this diagram.

Now consider, using Figure 1, the dynamically efficient optimal solution of the \( N \)-period depletion problem. The solution must intuitively result in,
as stated by Hotelling (1931), the present value of a unit extracted [being] the same in all periods if there is to be no gain from shifting extraction among periods. (Devarajan and Fisher 1981)

That is

\[ PVMNB_1 = PVMNB_2 = \ldots = PVMNB_t = \ldots. \]

the simple but important equal marginal result underlying all optimization theory. The optimal solution is obtained by charging a uniform present value marginal user cost \( \lambda \) (denoted \( \lambda \)) as a markup above \( MC_e \) in all periods, which is just big enough to produce a combined cumulative extraction exactly equal to the stock size. This markup, \( \lambda \), is illustrated in Figure 1 as the horizontal \( \lambda \) line a distance of \( \lambda \) units above the bottom axis. Its intersection with the \( PVMNB \) curve determines the optimal extraction rate \( q \) in any period \( t \). Note also that because discounting has no impact in period 1, \( \lambda + MC_e \) is the optimal starting price, that is, the price in period 1.

The question is, however, how big does \( \lambda \) need to be to yield the optimal outcome, and how can it be determined? One approach would be to solve graphically by trial-and-error, varying the size of \( \lambda \) until the resource stock constraint is exactly satisfied. This is the graphical equivalent of the solution technique suggested by Tietenberg for the formal mathematical optimization approach to solving the problem (Tietenberg 1992, appendix to Chap. 6), and suffers the same problem as the Pearce and Turner solution method: it is tedious. A simpler, less tedious, solution method can, however, be used. Define the last period of extraction as period \( T \). To ensure a zero extraction rate for period \( T + 1 \), the vertical intercept of \( PVMNB \) must therefore equal \( \lambda \). Thus, from equation (4), we see that

\[ \lambda = \frac{(a - MC_e)}{(1 + r)^T}. \]  

(5)

For each period \( t = 1, \ldots, T \), the level of extraction \( q_t \) is determined by \( PVMNB_t = \lambda \), thus from equation (4)

\[ q_t = \frac{(a - MC_e)}{b} - \frac{\lambda}{b}(1 + r)^{t-1}. \]  

(6)

In addition, total extraction must equal the resource stock size \( Q \)

\[ Q = \sum_{t=1}^{T} q_t. \]  

(7)

Substituting equation (5) into equation (6), and (6) into (7) yields

\[ Q = \frac{(a - MC_e)}{b} \left[ T - \frac{\sum_{t=1}^{T} (1 + r)^{t-1}}{(1 + r)^T} \right]. \]  

(8)

The solution method is now straightforward. The value of all variables in equation (8) are known, except that of \( T \), which can be easily and quickly solved by numerical iteration using a spreadsheet, or even a calculator (with or without the aid of compound interest tables). Once \( T \) is known, this implies \( \lambda \) (from equation (5)). Figure 1 can then be immediately drawn with the correct \( \lambda \) line, thus yielding the optimal values \( q_1, \ldots, q_T \) (which can also be determined directly from equation (6)).

The above discussion has considered time in discrete units. The analysis can, however, be just as easily undertaken for the continuous time case. There, expression (3) must be replaced by \( PVMNB_t = MNB_t e^t \), yielding \( \lambda = (a - MC_e) e^{\lambda T} \), and (by integration)

\[ Q = \frac{(a - MC_e)}{b} \left[ T - \frac{1}{r} \left( 1 - \frac{1}{e^{rt}} \right) \right]. \]  

as replacements for expressions (5) and (8). The value of \( T \) can then once again be solved simply by numerical iteration of this last expression, thus implying \( \lambda \), and hence a solution.

Figure 1, along with the simpler solution technique discussed above, is one way in which multiperiod analysis can be improved. An even more comprehensive approach is to integrate this into Pearce and Turner's 4-quadrant framework. Figure 2 presents a modified version of Pearce and Turner's diagram. Their northwest
2. This is the result obtained from solving the optimization problem of maximizing economic surplus.

4. Note that in the northwest quadrant time is measured through the labeling of the axes, in which Hotelling's famous "r per cent rule" applies, remains an important aspect in the economics of natural resources. Graphical presentations of this article have argued that to obtain a full understanding of the nature of optimal depletion, one must use a multiperiod analysis as a complement to the more common two-period analysis, and that existing multiperiod graphical presentations have some deficiencies. These are overcome by an alternative approach proposed in the article which: operates in a general N-period context; illustrates the full intertemporal impact of discounting; and, uses a simpler solution method for deriving optimal levels of extraction and prices over time.

CONCLUDING COMMENTS

The analysis of the problem of optimal depletion of exhaustible resources under the simple assumptions of intertemporally stable demand and constant marginal extraction costs, in which Hotelling's famous "r per cent rule" applies, remains an important aspect in the economics of natural resources. Graphical presentations of this analysis play an important role, particularly from a pedagogical perspective. This article has argued that to obtain a full understanding of the nature of optimal depletion, one must use a multiperiod analysis as a complement to the more common two-period analysis, and that existing multiperiod graphical presentations have some deficiencies. These are overcome by an alternative approach proposed in the article which: operates in a general N-period context; illustrates the full intertemporal impact of discounting; and, uses a simpler solution method for deriving optimal levels of extraction and prices over time.

NOTES

1. Note also that Pearce and Turner have incorrectly shown the extraction time path as being convex to the origin, rather than concave (as it should be for the linear demand curve case that they consider).

2. This is the result obtained from solving the optimization problem of maximizing economic surplus subject to the finite resource stock availability constraint (see, for example, Tietenberg 1992, appendices to chaps. 2 and 6).

3. The marginal user cost represents the difference between the full marginal social cost of extraction and the private marginal extraction cost, MC. The difference exists because extraction today prevents extraction tomorrow, thus generating an opportunity cost known as the marginal user cost.

4. Note that in the northwest quadrant, time is measured through the labeling of the axes, whereas in the other quadrants, time is on one or more of the axes.