future, and hence the less likely we are to honour the idea of conserving the natural capital stock. If high discount rates lead to a depletion of the capital stock, then sustainability is jeopardised.

For all these reasons, then, discounting is important. Many environmentalists, and some economists, even regard it as immoral simply because it does appear to be inconsistent with the ideas of conservation and sustainability. Yet discounting turns out to be an everyday occurrence. We therefore need to understand its basis.

14.2 THE RATIONALE FOR DISCOUNTING

The existence of interest rates explains discounting. One pound in year 1 would accumulate to £(1 + r) in year 2 if the interest rate is r per cent (r is typically expressed as the corresponding decimal, e.g. 5 per cent would be 0.05, 12 per cent would be 0.12, and so on). Looked at from the standpoint of year 1, we can ask the question: 'How much is £1 in year 2 worth to us in year 1?' The answer will be that it is worth £1/(1 + r), for the simple reason that if we had this sum in year 1 we could invest it at r per cent and obtain in year 2

\[
\frac{\£1}{(1 + r)} = \£1
\]

In the same way, we see that £1 in year 3 can be expressed as a value to us in year 1 as follows:

\[
\frac{\£1}{(1 + r)^2}
\]

since in year 3

\[
\frac{\£1}{(1 + r)^3}
\]

We now have the general formula for discounting. A benefit B in any year t can be written as B_t, and from the above procedure we know that this benefit will have a value to us in year 1 of

\[
\frac{B_t}{(1 + r)^t}
\]

Notice the procedure whereby we look at future benefits (and costs; the procedure is the same) from the standpoint of the present. This is why expressions such as the one above are called present values. The procedure for finding a present value is known as discounting and the rate at which the benefits or costs are discounted is known as the discount rate.

How then do positive interest rates arise? There are two underlying reasons for positive discount rates. First, people discount the future because they simply prefer their benefits now to later. We say they have time preference: they are impatient. The underlying value judgement in welfare economics is that people’s preferences matter — they should count in whatever social decision-rule we devise. The alternative to letting preferences count is to override them, to say that we know best what is good for other people. This overriding of preferences is something that societies commonly do, but it is clearly only to be done when there are very good reasons for it. If we accept that preferences matter, we are logically obliged to accept that people’s preferences for the present over the future must be allowed to count as well.

The second source of interest rates is the productivity of capital. The basic observation about capital is that if we divert some resources for investment (capital formation) rather than consumption, those resources will be able to yield a higher level of consumption in a later period than if we consumed them now. Clearly, it is worth waiting for these extra future benefits provided the cost in terms of impatience (the time preference cost) is less than the future benefits. We therefore see that there is a strong link between capital productivity and time preference: we are not likely to undertake investment unless the future benefits outweigh the time preference interest rate.

The STPR

If we now translate these two ideas to the level of society, we can argue that positive interest rates have two sources. The first is society’s time preference rate. This we call the social time preference rate (STPR). The STPR will reflect more than the underlying level of impatience which is known as pure time preference. It is likely also to reflect a social judgement to the effect that since future societies are likely to be richer than current ones, an extra $1 or £1 of benefit to them is worth less, has less utility, than $1 or £1 to the current society. This can be formalised by saying that we should discount the
future because of diminishing marginal utility of consumption. In fact we can present a formula for the STPR as follows:

\[ e = \text{STPR} = c \cdot e + p \]

where \( c \) = the rate of growth of real consumption per capita, \( e \) = the elasticity of the marginal utility of consumption function (see below) and \( p \) = the pure time preference rate of interest.

We do not derive the formula here (see Further Reading to this chapter for sources). The measure \( e \) shows the relationship between the utility that we think is derived from extra units of consumption, and, for analytical convenience, the relationship is expressed as an elasticity, i.e., the percentage change in utility that would arise from a percentage change in consumption. The component \( ce \) in the formula thus accounts for the idea that as future societies are likely to be richer we should attach less weight to their gains, i.e., we should discount those future gains. The component \( p \) reflects impatience. In each case we would get a percentage, so that adding the two together will give us \( e \) in a percentage form.

**SOC**

The second source is the productivity of capital. This is easier to understand because all we need to do is to express the future flow of benefits obtainable from investment as a flow of consumption. If we invest \( 100 \) units and get back \( 110 \) in consumption in the future, the net productivity of capital is \( 10 \) units; \( 10/100 = 10 \) per cent. Suppose that this 10 per cent covers all the benefits so that we can ignore any external effects, then 10 per cent is a social rate of return to capital investment. If we are considering a use of resources that yields us \( 8 \) per cent while we know that by investing elsewhere we could get 10 per cent then the proper decision is not to invest in the 8 per cent project - we could do better elsewhere. The 10 per cent is thus the social opportunity cost; or its SOC, of the project under consideration.

We can show the underlying analytics with the aid of Figure 14.1. This shows consumption in two years, \( t \) and \( t + 1 \). The function \( TT' \) is a transformation function, or production possibility curve. It shows the possible configurations of production between two years. It says, for example, that if resources are wholly devoted to year \( t \), output will be \( OT \). If they are wholly devoted to year \( t + 1 \) output would be \( OT' \). Notice that \( OT' \) is greater than \( OT \). This is intuitively acceptable because devoting resources to production in year \( t + 1 \) means, given that we have measured the transformation in terms of units of consumption, that \( OT' \) can only come about by investing all the resources that would have been consumed in year \( t \). That is, if the economy is at position \( T \) it means that \( OT \) was invested in year \( t \) and the consumption goods resulting are \( OT' \) in year \( t + 1 \).

Also shown in Figure 14.1 is an indifference curve. This is a social indifference curve indicating the combinations of consumption in period \( t \) (\( C_t \)) and consumption in period \( t + 1 \) (\( C_{t+1} \)) between which society is indifferent. This is denoted by \( SS' \).

Clearly, society will be in an optimal position if it located at point \( X \), for then society is able at this point to climb on to the highest possible social indifference curve given the constraints set down by the function \( TT' \). In fact we can find out just how much investment and consumption occur if the economy settles at \( X \). We can read off \( C_t \) and \( C_{t+1} \) immediately and we see that these are given by \( C_t \) and \( C_{t+1} \). But \( TT' \) embodies the investment that also takes place, i.e., the difference between \( C_t \) and \( OT \) must be the level of real investment in year \( t \), \( I_t \). In turn we see that it is \( I_t \) that generates the consumption...
level $C_{i+1}$. Using these ideas we can establish some important equations.

First, observing the level of investment $I_i$, we can see that

$$\frac{\Delta C_{i+1}}{I_i} = \frac{Y \bar{C}_i + XY}{I_i} = \frac{Y \bar{C}_i + XY}{I_i}$$

But $Y \bar{C}_i = I_i$ because $Y \bar{C}_i$ is drawn by constructing the $45^\circ$ line $YT$. Hence

$$\frac{\Delta C_{i+1}}{I_i} = 1 + \frac{XY}{I_i}$$

The first expression in (14.2) is the gross productivity of capital, and

$$\frac{XY}{I_i}$$

is the net productivity of capital, or its internal rate of return (or marginal efficiency of capital). Yet this latter concept is precisely the discount rate introduced as the first of the two alternatives, i.e. the interest rate in the economy, $r$. Moreover, if we make $I_i$ very small in Figure 14.1, we can see that $\Delta C_{i+1}/I_i$ measures the slope of $TT'$ (the tangent of angle $\bar{C}_i \bar{X}/\bar{C}_iT$ and $\bar{C}_i \bar{X} = O \bar{C}_i + 1$). Hence we can rewrite (14.2) as:

$$\text{Slope of } TT' = 1 + r$$

where $r$ is now the marginal rate of return on capital.

Turning our attention to $SS'$ we can proceed in a similar way. Consider points $J$ and $K$ in Figure 14.1. These are points on the same social indifference curve, so that the utility lost by moving from $K$ to $J$ would be $\Delta C_i M U_i$, i.e. the change ($\Delta$) in $C_i$ multiplied by the marginal utility associated with $C_i$. The utility gained would be $\Delta C_{i+1} M U_{i+1}$. Since $J$ and $K$ are on the same indifference curve, we can write

$$-\Delta C_i M U_i = \Delta C_{i+1} M U_{i+1}$$

and hence

$$\frac{-\Delta C_{i+1}}{\Delta C_i} = \frac{MU_i}{MU_{i+1}}$$

The slope of $SS'$ is simply the ratio of the two marginal utilities of consumption (which is what we would expect from our knowledge of indifference curves in general).

Now, as we move along $SS'$ in the direction $K$ to $J$, society will tend to require more and more $C_{i+1}$ to compensate for a unit loss of $C_i$. Very simply, $\Delta C_{i+1}/\Delta C_i > 1$. Hence we now have, from (14.5),

$$\frac{MU_i}{MU_{i+1}} > 1$$

Writing the excess of this ratio over unity as $s$ we have

$$\frac{MU_i}{MU_{i+1}} = 1 + s = \text{Slope of } SS'$$

We now define $s$ as the social rate of time preference.

The discussion so far suggests that there are two possible sources for the discount rate, $r$. It could be SOC or STPR. In a world in which there are no taxes and where there are perfectly functioning capital markets, it turns out that $SOC = STPR$: we would not have to worry about which approach we took. In a less than perfect market situation, however, a choice has to be made. A voluminous literature exists on this issue, but it does not concern us here. Suffice it to say that some people argue for SOC, some for STPR and some for a sort of average of the two.

Whatever, the school of thought in question, however, the discount rates that emerge are all positive. That is, the literature has a consensus that the fact that people prefer the present to the future and the fact of capital productivity imply positive discount rates. We need to see whether this conclusion is likely to be modified by criticisms advanced by environmentalists.

14.3 A CRITIQUE OF DISCOUNTING

Pure time preference

The objections to allowing the component $p$ (reflecting impatience) to influence the discount rate are several. First, economists would point out that individual impatience is not necessarily consistent with maximising an individual's lifetime welfare maximisation. The proof is complex but is a variant of the idea expressed by a number of economists that 'impatience discounting' is fundamentally irrational. It can lead to decisions which are incompatible with long-run welfare. Second, what individuals want carries no necessary