Math 152 Mock Exam 1

The ease with which your solutions can be followed is as important as the final answer. Be sure your plan and the order of your steps are clear to the reader.

DO NOT USE CALCULATORS.

(Evaluate $e^0=1$, $\cos(\pi/4)=\sqrt{2}/2$ etc. Leave $\cos(5), \sqrt{5}$ etc. unevaluated.)

You may use a 3 by 5 card. You may write on both sides of the card, but not on the edges.

1. You will be asked to derive one of the following derivative formulas and show all the important steps:

$$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2},$$

or $\frac{d}{dx}\ln(x) = \frac{1}{x}$.

2. Find the following integrals by guess and check, substitution, expansion, or other elementary means. (You will be given 4-6 integrals similar to those found in section 7.1 of our text on pp. 296-297, 3 - 40.)

3. Compute the following integrals. (Integration by parts can help find them, but any correct method is acceptable.) I will look for problems similar to homework problems from section 7.2. In particular I will look for examples of PPD, T&S, O&I, and Inv. For example this would be a possible set.

a. $\int \arctan(x)dx$

   Ans. ________________________________.

b. $\int \cos^2(t)dt$

   Ans. ________________________________.

c. $\int e^x \cos(y)dy$
4. Use partial fractions, factoring or completing the square to compute the following. Show the steps of the method. (Inspection or guess-and-check is not acceptable.)

a. \[ \int \frac{1}{v^2 + 4v + 3} \, dv \]

b. \[ \int \frac{1}{9 + x^2} \, dx \]
5. Compute the following improper integrals. Give their values if they converge. If they diverge, say so and clearly show why the limit doesn’t exist.

   a. \( \int_{1}^{\infty} \frac{1}{\sqrt{x}} \, dx \)

   b. \( \int_{1}^{\infty} \frac{x}{4 + x^2} \, dx \)

   Ans. ________________________________.

6. Let \( f(x) \) be a function to be numerically integrated on the interval from \( x = -3 \) to \( x = 3 \). Let \( n \) be the number of subdivisions in the interval. Answer the following.

   a. Write an expression for the width, \( \Delta x \), of each subinterval, in terms of the information given above.

      ans.____________________

   b. Write the expression for the right endpoint of the \( i^{th} \) subinterval, \( r(i) \).

      ans. ______________________________

   c. Write an expression for \( m(i) \), the midpoint of the \( i^{th} \) subdivision, in terms of the information given above.

      ans____________________________

   d. Write the expression for the Midpoint Riemann Sum of the above function.

      ans.____________________________

   e. The function whose graph is shown here is to be numerically integrated. List the following from smallest to largest: (Each method uses the same interval and subdivision.)

   \[ L = \text{Left Riemann Sum}, \quad R = \text{Right Riemann Sum}, \quad M = \text{Midpoint Riemann Sum}, \quad T = \text{Trapezoid Rule}, \quad E = \text{Exact Value}. \]

      ans.________________________________________

7. Integrate the following by any appropriate method. Clearly indicate the method.

   a. \( \int \cot(x) \, dx \)
b. \[ \int \frac{1}{\sqrt{1 + 9x^2}} \, dx \]

8. Let \( f(x) = \frac{1}{x} \). Show with a clear computation whether \( \int f(x) \, dx \) converges or diverges.