Math 152 Exam 3 (Diff Eq)

Name _____________________

The ease with which your solutions can be followed is as important as the final answer. 
**Be sure your plan and the order of your steps are clear to the reader.**

(Evaluate $e^1 = 1$, $\cos(\pi/4) = \sqrt{2}/2$ etc. Leave $\cos(5)$, $E$, etc. unevaluated.)

You may use a 3 by 5 card. You may write on both sides of the card.

You may use calculators, but calculator answers are not acceptable substitutes for clear work and/or explanations.

Give reasons whenever it will help the reader follow the work.

1. Use Euler’s method with initial condition $y(-0.5) = 0.5$ and step size $\Delta x = 0.25$ to find a numerical solution to $\frac{dy}{dx} = xy$ from $x = -0.5$ to $x = 0.5$. Clearly show the Euler Table.

   (10pt)

2. For each of the following differential equations, list all functions from the given list that satisfy it.

   a. (6pt) $\frac{dy}{dx} = \frac{y}{x}$
      ans._________________
      i. $y = 0$
      ii. $y = x^2$
      iii. $y = 2x$
   b. (6pt) $\frac{dy}{dx} = 2y$
      ans._________________
      iv. $y = e^{2x}$
      v. $y = 2e^x$

3. (3pt. each) Match each slope field to the differential equation which it represents. If there is no slope field matching an equation, write “None”.

   a. $\frac{dy}{dx} = 2y$   ans._________________
   b. $\frac{dy}{dx} = \frac{y}{x}$   ans._________________
   c. $\frac{dy}{dx} = \frac{y}{x}$   ans._________________
   d. $\frac{dy}{dx} = y \sqrt{x}$   ans._________________
4. (10pt) Solve by separation of variables: \( \frac{dy}{dx} = \frac{x}{y} \quad \text{y(0) = 4} \)

5. The growth of a certain animal population is governed by the equation:

\[
\frac{1000}{P} \frac{dP}{dt} = 100 \quad P
\]

where P(t) is the number of individuals at time t.

a. (5pt) Sketch a graph of \( \frac{dP}{dt} \) as a function of \( P \). Show clearly where \( \frac{dP}{dt} \) is zero, positive and negative.
b. (5pt) Sketch a graph of the slope field of the differential equation.

\[ \begin{array}{c}
\text{p} \\
\hline
\hline
\text{t}
\end{array} \]

c. (5pt) The initial population is 200 individuals. Will the population ever exceed 200? Explain clearly.